Note: Nearly all the problems below are of the “how many...?” variety. Please provide two answers to each question, in addition to showing your work: the formula you used to compute the answer, and the actual answer. For instance, if I asked you, “how many anagrams does the word angriest have?” please write down both:

\[ 8! \]

and:

\[ 40,320 \]

on your answer sheet. I want you to give me the former in order to enable possible partial credit for wrong answers, and I want you to give me the latter so you get some appreciation for the magnitude of the number involved.

Sometimes it will be convenient to use scientific notation, in order to express large numbers as a number times a factor of ten. For instance, if I asked you, “how many anagrams does the word antidisestablishmentarianism have?” you should write down both 28! and \[ 3.048883 \times 10^{29} \] as answers.

1. In the WayCool™ programming language, each variable is given a name of up to 12 characters. The variable name must begin with an upper or lower case English letter, but the rest of the characters can be letters, the digits 0-9, or underscores (“_”). How many possible variable names are there in WayCool™?
2. DES (Data Encryption Standard) was a cipher algorithm used in the 1970’s to encrypt data during transmission. A sender used a 56-bit key to scramble their message, and sent the scrambled message to a recipient. The recipient, with a copy of the 56-bit key, would then unscramble it to retrieve the message. (Note that a “bit” is a binary digit – either a 0 or a 1. A 56-bit key, therefore, is a string of 56 bits, such as 01110011011001100100011010000110100011000110001100011.)

a. How many possible DES keys are there?

b. Suppose an Evil Individual™ wanted to intercept a communication and decipher it for some vile purpose. How long would it take, on average, for him to break the code by brute force? (i.e., trying every possible key until he finds the one that works?) Assume the Evil Individual™ is using a really fast computer with custom hardware that can try 1 billion keys per second. Express your answer in years.

c. Triple-DES is a newer algorithm that basically works the same way that DES does, but it uses a 168-bit key. How long would the Evil Individual™ need, on average, to brute force break this code? Express your answer in years.
3. As everyone knows, Ben & Jerry’s ice cream is the most delectable substance known to humankind. Ben & Jerry’s comes in many different “recipes” – such as New York Super Fudge Chunk, Cherry Garcia, and Make Mine Vanilla. Each recipe has one base flavor (New York Super Fudge Chunk’s is chocolate) and any number of mixins (NYSFC has fudge flakes, peanuts, walnuts, and almonds OH MY GOSH it’s so good; Cherry Garcia has fudge flakes and cherries; while Make Mine Vanilla has no mixins at all.)

a. There are 15 different base flavors and 27 different mixin types. How many different Ben & Jerry’s recipes are possible?

b. Some people like smooth stuff, but I like crunchies in my ice cream. How many different Ben & Jerry’s recipes are possible that have at least two mixins?

4. Originally, computer engineers in the western world used one byte to represent each character in a text message. (A “byte” is a sequence of 8 bits, like 11101001.) They invented a code called the ASCII code which (rather arbitrarily) assigned 01000001 to the capital letter ’A’, 00111101 to the plus sign ’+’, 00110111 to the digit ’7’, etc.

Trouble is, although there are enough different combinations of 8 bits to represent all the letters in the English alphabet (and numbers, and keyboard symbols, etc.) the world doesn’t just speak English. There are also Chinese and Korean characters, the Arabic and Hebrew alphabets, etc.

Question: suppose there are 100,000 distinct symbols in common use on planet Earth. (A reasonable guess.) How many bits would be needed to represent a character, if a distinct bit pattern was desired for each different symbol?
5. A room for a dinner party is prepared with ten cozy tables-for-two.

a. Ten gentlemen and ten ladies attend the party. Each gent finds a lady to sit down with. How many different seating arrangements are possible? (A “seating arrangement” is a set of pairings of all the individuals at the party. For example, “Steve with Janice, Fred with Sally, Bob with Cindy, Stanley with Rebecca, ..., and Filbert with Jezebel” is one seating arrangement. Note that it doesn’t matter which particular table Bob and Cindy sit at, or which of the two seats at that table Bob sits in; if Bob and Cindy are together, that’s all that matters.)

b. Twenty ladies attend the party. Each lady finds another lady to sit down with. How many different seating arrangements are possible? (A “seating arrangement” is a set of pairings of all the individuals at the party. For example, “Jane with Beverly, Jasmine with Esther, Beyoncé with Kim, ..., and Ke$$ha with Hermione” is one seating arrangement. Note that it doesn’t matter which particular table Jane and Beverly sit at, or which of the two seats at that table Jane sits in; if Jane and Beverly are together, that’s all that matters.)

6. Biff is watching his weight. He’s going to eat three square meals a day, and no snacking in between! For breakfast, he can choose between oatmeal with wheat germ, nonfat yogurt, or bacon & eggs. For lunch, he can have a whole wheat muffin, a lean turkey sandwich, a hamburger with french fries, or fried chicken. For dinner, his choices include country-fried steak lathered in gravy, steamed brussel sprout soufflet, or boiled organic tofu ramen noodles.
a. How many possible menus does Biff have for each day? (A “menu” is comprised of a breakfast choice, a lunch choice, and a dinner choice.)

b. How many possible daily menus does he have in which he “splurges” at least once during the day? (i.e., eats a high-fat, low-fiber meal?) Note: if you have any doubts about which of the above items are high-fat/low-fiber, please ask.

7. Johnny Davies is a student in Mrs. Koningsbauer’s class at Spotswood Elementary. Mrs. Koningsbauer tells all the fifth-graders that they have to work on their spelling words each and every day for homework. They have a set of 9 different activities they can choose from each day (shuffle sort, spell the words while laughing, make a word search, etc.) and although they can choose any one activity to do on a given day, they cannot choose the same activity more than once in a week. How many different sets of activities are possible for Johnny to choose each week? (It doesn’t matter which activity he does on which day.) By the way, elementary school students have to do homework only 5 days a week (they get the weekend off.)
8. It’s almost class registration time at UMW! How many different possible schedules could a student have next semester, assuming he or she took **either 5 or 6 classes**? State your assumptions and how you get your data. Note that a “schedule” is a set of classes a student might take during fall semester. It doesn’t matter what time the classes are, nor what room they’re in, nor which section of a course they’re in – all that matters is the set of courses that will end up on their transcript at the end of the semester.

9. In NCAA basketball, a “Final Four” is a set of four teams who will compete in late March for the National Championship. For instance, “Duke, Kentucky, North Carolina, and Gonzaga” would be one possible “Final Four.” A different Final Four would be “Duke, Louisville, Kentucky, and Kansas.” Note that it’s not a different Final Four unless there’s at least one different team in it; *i.e.*, “Duke, Louisville, Kentucky, and Kansas” is the same Final Four as “Louisville, Kentucky, Kansas, and Duke.”

   a. Before the season begins, how many different NCAA Final Fours are possible? (Assume there are 346 NCAA teams.)

   b. In early March, the NCAA will publish the 64-team “bracket,” which shows the top 64 chosen teams, and which will play each other on their way to the Final Four. (An example bracket can be found at: [http://cs.umw.edu/~stephen/cpsc125/bracket.pdf](http://cs.umw.edu/~stephen/cpsc125/bracket.pdf). Ignore the “play-in games” part.) After they announce the bracket, but before any games are played, how many different NCAA Final Fours are possible?