Homework #1 – Sets and relations
Possible experience: +40XP
Due: Friday, Sept. 16th, 11:59pm

(This homework set is based on episodes II and III of [http://allthemath.org](http://allthemath.org) and chapters 2 and 3 of [A Cool Brisk Walk](http://allthemath.org).

Use these sets in the following problems:

- \( A = \{ \text{Jar Jar Binks, Barney, Mary Washington, Martha Washington} \} \)
- \( B = \{ 4, 6, 7 \} \)
- \( C = \) the set of all non-prime positive integers
- \( F = \{ l \mid l \text{'s husband is or was president of the United States} \} \)
- \( G = \) the set of all African-Americans
- \( H = \{ q \mid q \text{ is prime and } 1900 < q < 2100 \} \)
- \( J = \) the set of years in which the New York Yankees won the World Series
- \( M = \) the set of all male players on the Washington Mystics
- \( S = \) the set of years in which the Denver Broncos won the Superbowl
- \( T = \) the set of years in which the Kansas City Chiefs won the Superbowl
- \( W = \) the set of all females
- \( U = \) the set of all colleges in the University of Mary Washington
- \( \mathbb{N} = \) the set of natural numbers
- \( \Omega = \) the set of absolutely all possible things

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1. Google can help for problems like this one.
2. And this one.
3. Ditto.
4. If you don’t know what the UMW “colleges” are, Google can help here too. Hint: one of them is “The College of Arts and Sciences.”
Questions:
1. Is 2000 ∈ H?
2. Is Barack Obama ∈ F?
3. Is Beyoncé ∈ W?
4. Is F ⊆ W?
5. Is Ĺ ⊆ Ĺ?
6. What is B ∩ H?
7. What is B ∩ C?
8. What is S ∩ J?
9. What is H ∩ S ∩ J?
10. What is H ∩ (S ∪ J)?
11. What is (G ∪ W) ∩ F?
12. What is P(A - F)?
13. What is | B |?
14. What is | P(U) |?
15. What is | P(U) ∩ P(B) |?
16. What is U × (B ∩ C)?
17. Does B ∪ N = N?
18. Does U ∪ N = N?
19. Does Ω ∩ R = R?
20. Does | G ∩ W | = | W |?
21. Does | J ∩ A | = | A |?
22. Does | F ∩ W | = | F |?
23. Does G ∪ H = T ∩ M?
24. Does G ∩ H = T ∪ M?
25. Do B ∩ C and B ∩ Ĺ form a partition of B? Explain.
26. Do W and Ĺ form a partition of Ω? Explain.
27. Let $M$ be the set $\{\text{Neo, Morpheus, Trinity}\}$. If we don’t allow the empty set to be a member of a partition, how many different partitions are there of $M$? List all of them.

28. Let $C$ be the set $\{\text{Gimli, Gandalf, Sauron, Saruman, Frodo}\}$. Let $T$ be an endorelation on the set $C$ with the following elements:
   - $(\text{Gimli, Gimli})$
   - $(\text{Gimli, Gandalf})$
   - $(\text{Gimli, Frodo})$
   - $(\text{Gandalf, Gandalf})$
   - $(\text{Gandalf, Gimli})$
   - $(\text{Gandalf, Sauron})$
   - $(\text{Saruman, Gandalf})$
   - $(\text{Gandalf, Frodo})$
   - $(\text{Frodo, Gimli})$
   - $(\text{Frodo, Gandalf})$
   - $(\text{Frodo, Frodo})$
   - $(\text{Sauron, Sauron})$

   a. Is $T$ reflexive? Why or why not?

   b. Is $T$ symmetric? Why or why not?

   c. Is $T$ antisymmetric? Why or why not?

   d. Is $T$ transitive? Why or why not?
29. Let $U$ be the set of all UMW students. For each of the following relations (defined as a subset of $U \times U$), specify whether (and why or why not) each of them is reflexive, symmetric, antisymmetric, and/or transitive.

a. $U \text{ isRoommateWith } U$
   - reflexive?
   - symmetric?
   - antisymmetric?
   - transitive?

b. $U \text{ isOneSchoolYearAheadOf } U$
   - reflexive?
   - symmetric?
   - antisymmetric?
c. $U \text{ hasKissedOnTheLips } U$
   • reflexive?
   • symmetric?
   • antisymmetric?
   • transitive?


d. $U \text{ hasReadMoreHarryPotterNovelsThan } U$
   • reflexive?
   • symmetric?
   • antisymmetric?
• transitive?

30. Let $R$ be the set of all recipes Betty likes, and $I$ be the set of all ingredients at her grocery store. Further, let criticalIngredient be a relation between $R$ and $I$ with the following members: $\{ (\text{PeachCobbler}, \text{peaches}), (\text{ChickenKiev}, \text{breadCrumbs}), (\text{PeachCobbler}, \text{flour}), (\text{Souffle}, \text{eggYolks}) \}$. Is criticalIngredient a function from $R \rightarrow I$? Why or why not?
31. Let \( \mathbb{N} \) be the set of natural numbers, and \( \mathbb{R} \) the set of real numbers. Further, let \( aSmidgenLessThan \) be a relation between \( \mathbb{R} \) and \( \mathbb{N} \) such that for every ordered pair \( p \in aSmidgenLessThan \), the second element is the least natural number that is greater than or equal to the first element. Is \( aSmidgenLessThan \) a function from \( \mathbb{R} \to \mathbb{N} \)? Why or why not?

32. Let \( A \) be the set of all airports in the world, and \( C \) the set of all three-letter airport codes. Further, define a function \( codeOf : A \to C \). For example, \( codeOf(ReaganNational) = DCA \), and \( codeOf(TorontoPearsonInternational) = YYZ \). Is \( codeOf \) injective? Surjective? Bijective? Explain.

33. Let \( P \) be the set of all UMW professors, and \( S \) the set of all UMW students. Further, define a function \( faveUMWProfessor : S \to P \). For example, \( faveUMWProfessor(RebeccaGraham) = DrFernsebner \), and \( faveUMWProfessor(ElizabethRamsey) = DrMatzke \). Is \( faveUMWProfessor \) injective? Surjective? Bijective? Explain.